

DE LA RECHERCHE À L'INDUSTRIE



SOPRANO : NEW SOLVER FOR PROGRAM ANALYSIS

Kickoff | F.Bobot

13 Janvier

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digiteo

list



SMT-solver: Big picture and CDCL

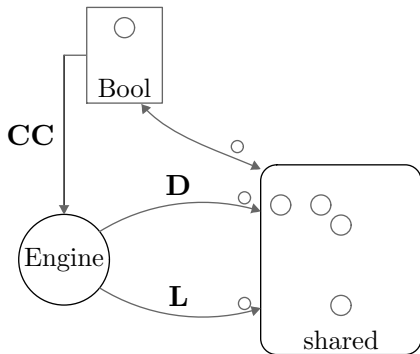


$$(A \vee B \implies C)$$

$$\wedge(C \implies D)$$

$$\wedge(D \wedge E \implies \neg A)$$

SMT-solver: Big picture and CDCL



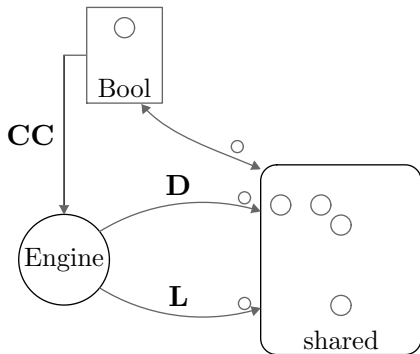
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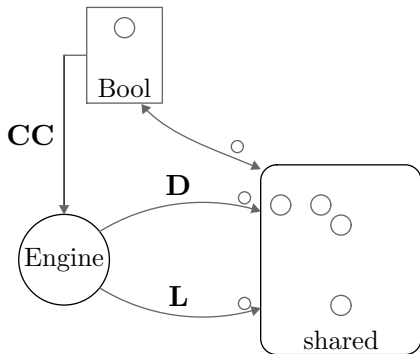
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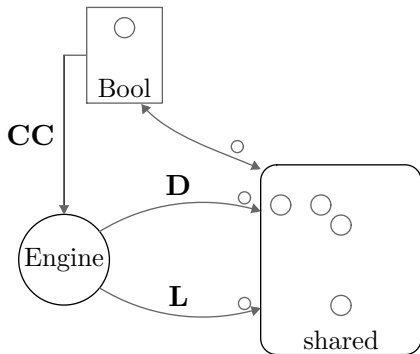
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⋮

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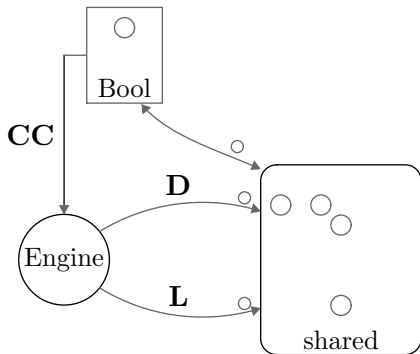
$$\wedge(D \wedge E \implies \neg A)$$

$$\frac{}{A_D}$$

$$\vdots$$

$$\frac{}{E}$$

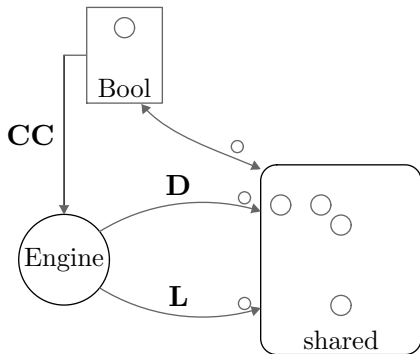
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SMT-solver: Big picture and CDCL



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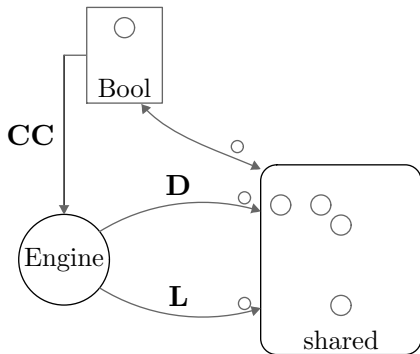
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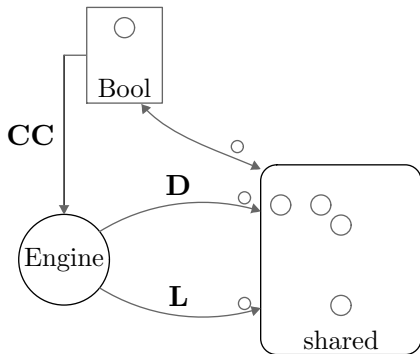
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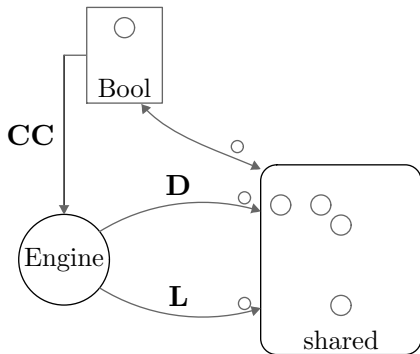
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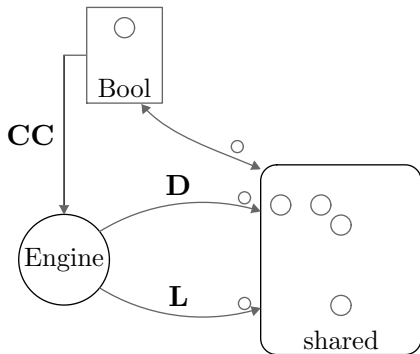
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—————
 $\neg D$
 C
 A_D
 \vdots
 E
 —————

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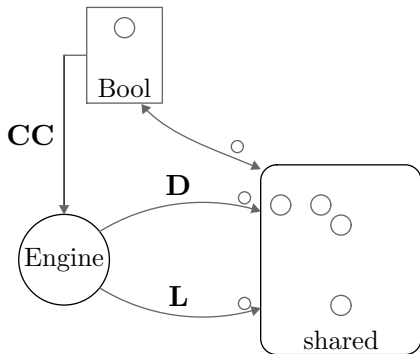
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SMT-solver: Big picture and CDCL



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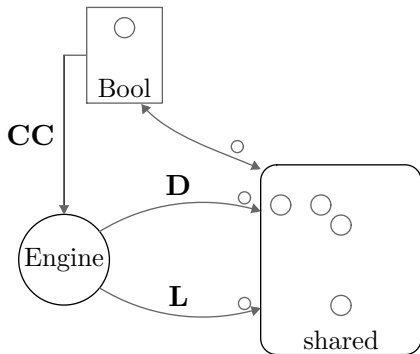
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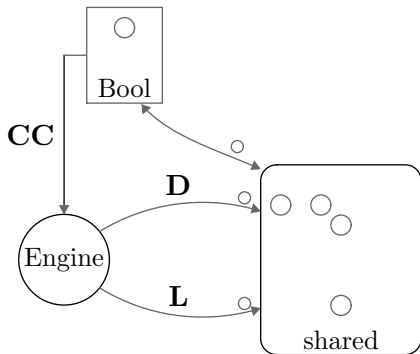
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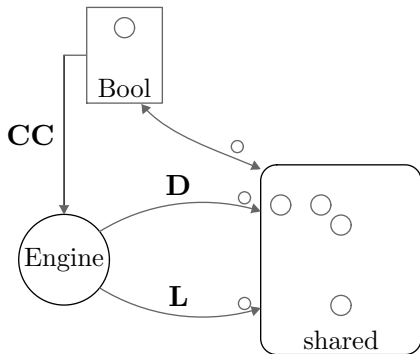
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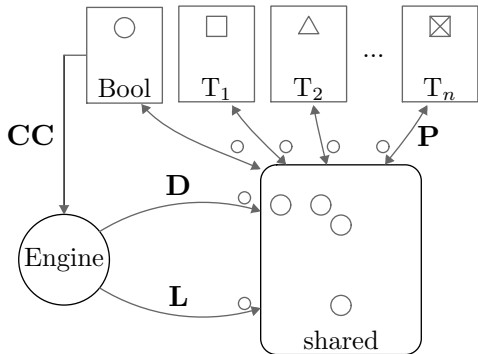


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 \end{aligned}$$

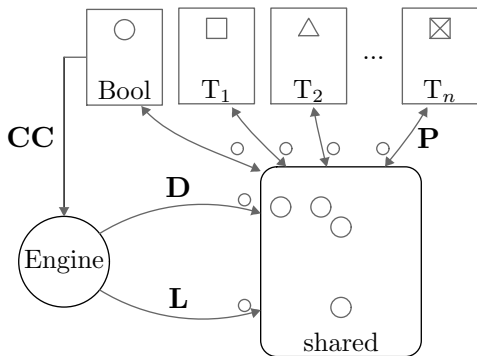
$$\begin{array}{c}
 \hline \\
 \\
 \\
 \hline
 \end{array}$$

$$\begin{array}{c}
 \neg A \\
 E
 \end{array}$$

SMT-solver: Big picture and CDCL



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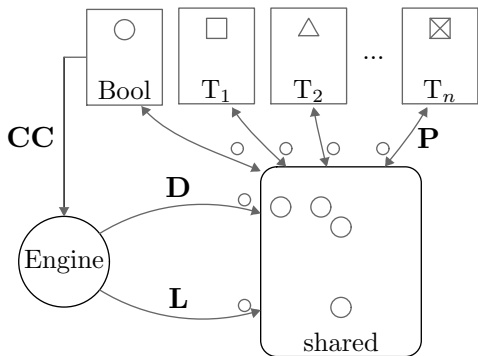


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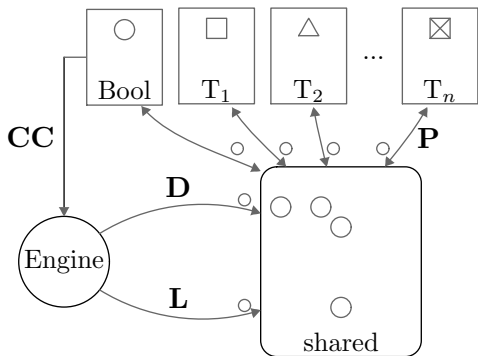
SMT-solver: Big picture and CDCL



$$\left(\begin{array}{l} 10 \leq x + t \vee 10 \leq y \\ \implies 10 \leq z \end{array} \right)$$

$$\wedge (10 \leq z \implies t < 0)$$

SMT-solver: Big picture and CDCL

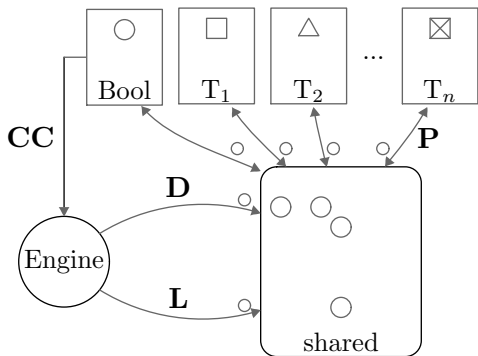


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$$\frac{}{(x \leq 0)}$$

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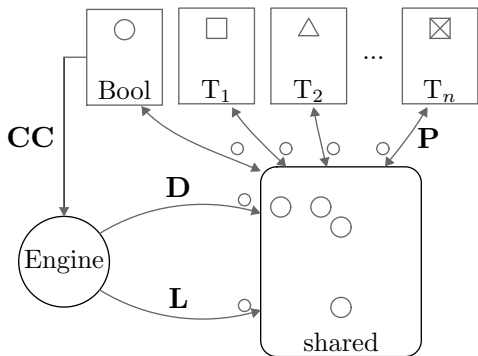
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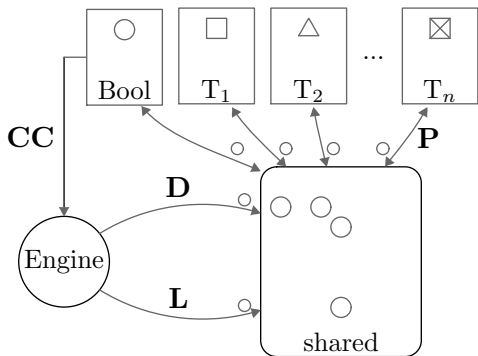
$$\wedge (10 \leq z \implies t < 0)$$

$$(10 \leq x + t)_D$$

$$\vdots$$

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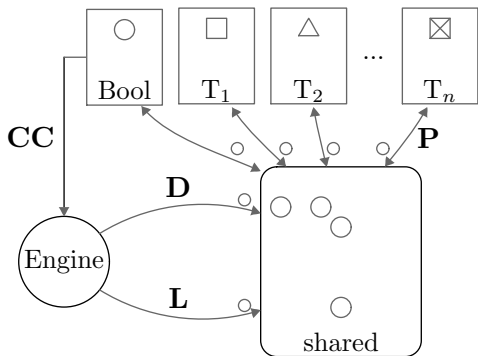


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$$\begin{array}{c} 10 \leq z \\ (10 \leq x + t)_{\mathcal{D}} \\ \vdots \\ (x \leq 0) \end{array}$$

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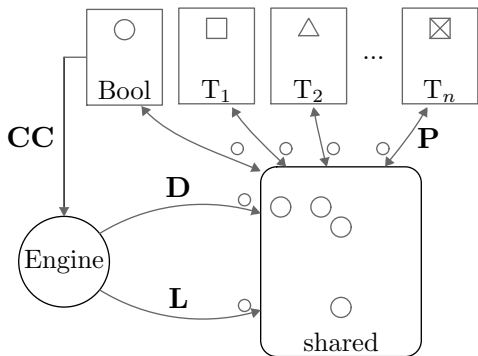
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$$\wedge \left(\begin{array}{l} t < 0 \wedge x \leq 0 \\ \implies -10 \leq x + t \end{array} \right)$$

$$\frac{\begin{array}{c} \neg(t < 0) \\ 10 \leq z \\ (10 \leq x + t)_{\mathcal{D}} \\ \vdots \\ (x \leq 0) \end{array}}{\quad}$$

SMT-solver: Big picture and CDCL



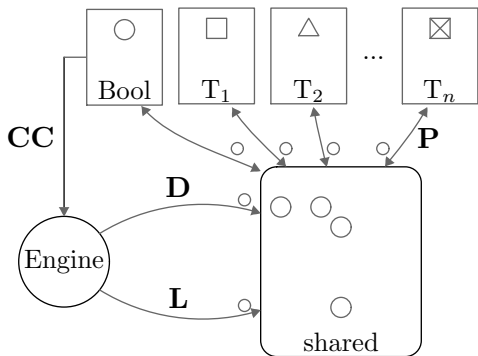
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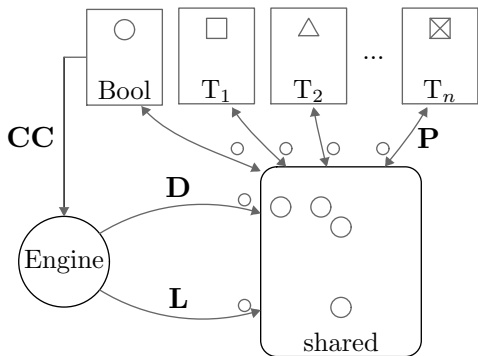
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$$\wedge (10 \leq z \implies t < 0)$$

$$\wedge (\neg x \leq 10 \vee \neg 10 \leq x + t)$$

$$(x \leq 0)$$

- Decision: an unset variable is added to the partial model
- Propagation: propagate values using the constraints
- Conflict: a constraint is unsatisfiable in the partial model
- Learning: compute a constraint which:
 1. is implied by the constraints
 2. is unsatisfiable in the partial model
 3. disallows the last decision by propagation

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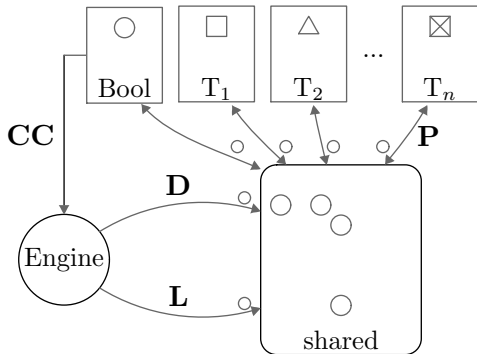
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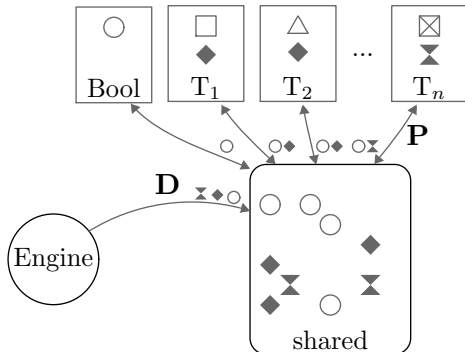
SMT-solver: Difficulties



- communication using only booleans
- split-on-demand:
 $x \leq 11 \vee 11 < x$
- learning: no new literals

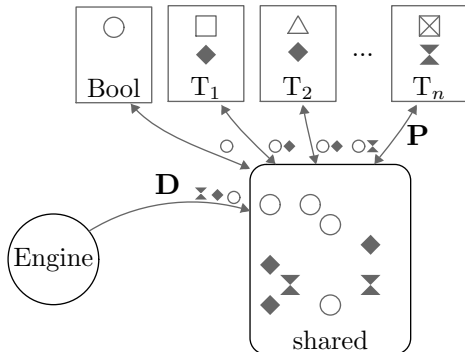


Big picture



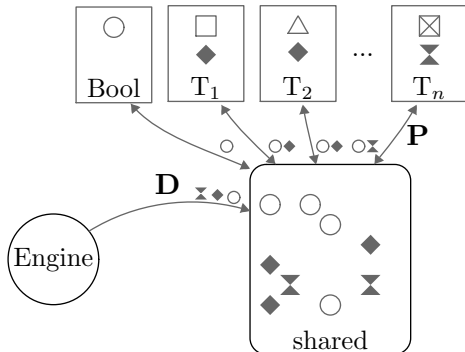
- Specific domains
- Precise and numerous propagator
- Easy model computation

Big picture



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Big picture



- Specific domains
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- Easy model computation

Propagation and domains

- arithmetic intervals: $\mathcal{D}_I(x) = \llbracket 1; 3 \rrbracket$ and $\mathcal{D}_I(x + y) = \llbracket 2; 4 \rrbracket$
implies $\mathcal{D}_I(y) = \llbracket -1; 3 \rrbracket$
- intervals and modulus: reduce product (AI)
- intervals and arrays:
select(x , store(store(store(const(4), y_3 , 3), y_2 , 2), y_1 , 1)) = t
 $\mathcal{D}_I(t) = \llbracket 1; 4 \rrbracket$
- bitvectors and arithmetic intervals: $\mathcal{D}_I(x) = \llbracket 0; +\infty[$ implies
 $\mathcal{D}_I(x \mid 31) = \llbracket 2^{32}; +\infty \rrbracket$
- floating-points

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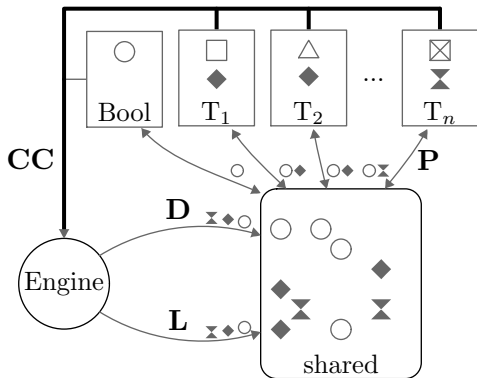
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- No backjumping
- No learning
- Costly propagation

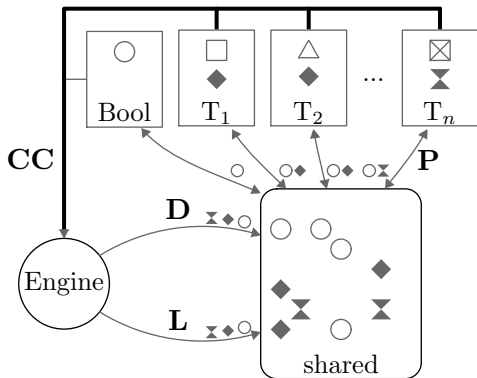
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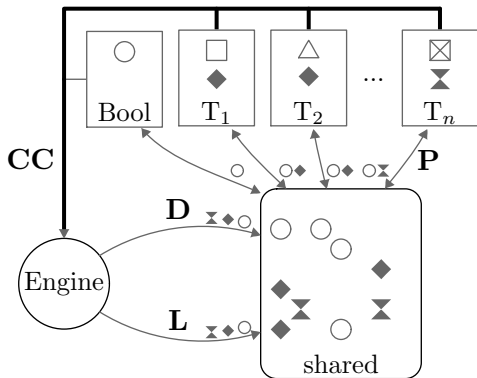




- Can decide on every variables
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Example: Arithmetic

$$10 \leq x - y \quad \wedge \quad 10 < x + y + z$$

1. Decide $x = 0$
2. Propagate using $x - y$
3. Conflict in $x + y + z$
4. Compute clause conflict
5. Propagate

	\mathcal{D}	<i>reason</i>
x	$] -\infty; +\infty[$	
y	$] -\infty; +\infty[$	
z	$] -\infty; 0]$...
$x-y$	$[10; +\infty[$	
$x+y+z$	$]10; +\infty[$	

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x	$\{0\}$	<i>dec</i>
y	$] -\infty; +\infty[$	
z	$] -\infty; 0]$	<i>...</i>
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z	$] - \infty; 0]$...
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y	$] -\infty; -10]$	$x - y$
z	$] -\infty; 0]$...
$x - y$	$[10; +\infty[$	
$x + y + z$	$]10; +\infty[$	

$$\begin{array}{r}
 \begin{array}{r}
 \text{dec} \\
 \hline
 0 \leq -x
 \end{array}
 \quad
 \begin{array}{r}
 \text{level0} \\
 \hline
 10 \leq x - y
 \end{array} \\
 \hline
 10 \leq -y
 \end{array}
 \quad
 \begin{array}{r}
 \text{dec} \\
 \hline
 0 \leq -x
 \end{array}
 \quad
 \begin{array}{r}
 \vdots \\
 \hline
 0 \leq -z
 \end{array}
 \quad
 \begin{array}{r}
 \text{level0} \\
 \hline
 10 < x + y + z
 \end{array} \\
 \hline
 20 < 0
 \end{array}$$

$$\begin{array}{r}
 \text{gen} \qquad \text{level10} \\
 \hline
 -x \leq -x \qquad 10 \leq x - y \\
 \hline
 -x + 10 \leq -y
 \end{array}
 \qquad
 \begin{array}{r}
 \text{gen} \qquad \text{gen} \\
 \hline
 -x \leq -x \qquad -z \leq -z
 \end{array}
 \qquad
 \begin{array}{r}
 \text{level10} \\
 \hline
 10 < x + y + z
 \end{array}$$

$$-z - 2x + 20 < 0$$

Example: Arithmetic

$$10 \leq x - y \quad \wedge \quad 10 < x + y + z$$

1. Decide $x = 0$
2. Propagate using $x - y$
3. Conflict in $x + y + z$
4. Compute clause conflict
5. Propagate

	\mathcal{D}	<i>reason</i>
x	$\{0\}$	<i>dec</i>
y	$] -\infty; -10]$	$x - y$
z	$] -\infty; 0]$...
$x - y$	$[10; +\infty[$	
$x + y + z$	$]10; +\infty[$	

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x	$[10; +\infty[$	$2x + z$
y	$] -\infty; +\infty[$	
z	$] -\infty; 0]$...
$x-y$	$[10; +\infty[$	
$x+y+z$	$]10; +\infty[$	
$2x+z$	$[20; +\infty[$	



```
git@git.frama-c.com:soprano/popop.git
```

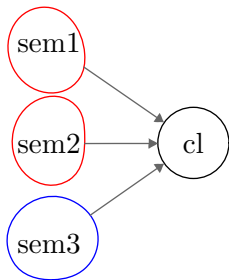
```
specifically written: 8417 code, 1409 comment
```

```
(reuse Why3 STDLIB, alt-ergo parser, smtlib2  
parser)
```



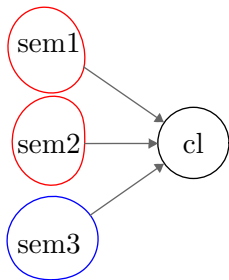
kind of semantic values:

- boolean formula $\neg(cl_1 \vee \neg cl_2 \vee cl_3)$
- rational polynomial $4cl_1 + 3cl_2 + 5$
- equality $cl_1 = cl_2 = cl_3 = cl_4$
- ite $ite(b, cl_1, cl_2)$
- uninterpreted symbols $App(cl_1, cl_2)$



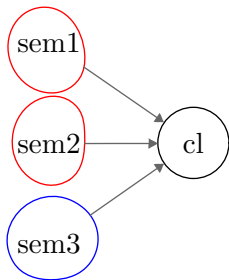
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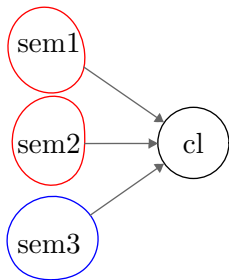
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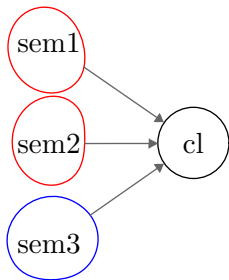
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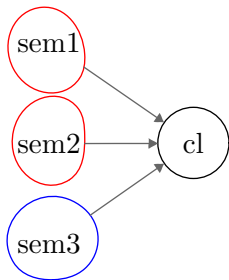
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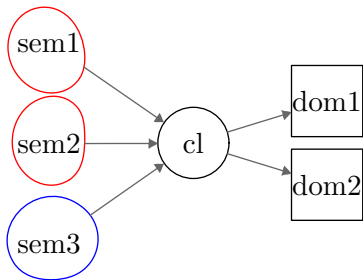
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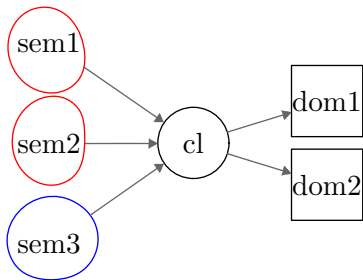
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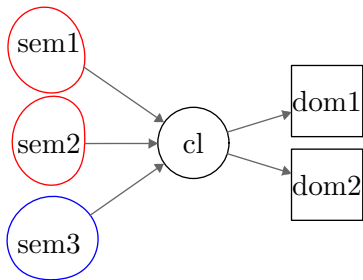
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- boolean $\{\top, \perp\}$
- distinct: $\{1; 9; 10\}$
- normalized polynomial
 $4c_1 + 3c_2 + 5$
- interval rational $[a; b]$



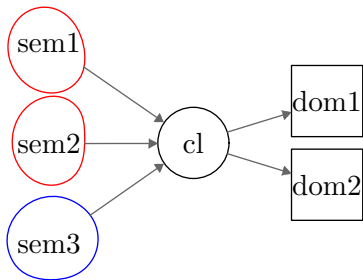
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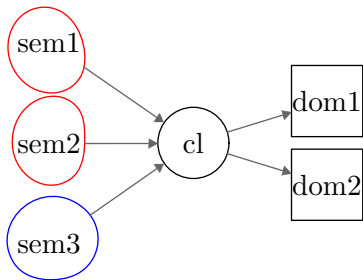
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src/types.mli:

```

module Sem: sig
  type 'a k
  val create_key: string -> 'a k
  ...
end

type 'a sem = 'a Sem.k

module type Sem = sig
  include Datatype

  val key: t sem
end

module RegisterSem (D:Sem) : sig
  include Datatype

  val index: D.t -> Ty.t -> t

  val cl: t -> Cl.t

  val sem: t -> D.t

  ...
end

```

```
src/solver.mli:
```

```
module type Dom = sig
```

```
  type t
```

```
  val merged: t option -> t option -> bool
```

```
  val merge:
```

```
    Delayed.t -> pexp ->
```

```
    t option * Cl.t (* c11 *) ->
```

```
    t option * Cl.t (* c12 *) ->
```

```
    bool (** true: c11 will be repr otherwise it is
```

```
    unit
```

```
  val print: Format.formatter -> t -> unit
```

```
  val key: t dom
```

Internal scheduler:

1. Set value of domains
2. Set semantic values
3. Immediate daemons (ex: polynomial normalization)
4. Merge domains: `merge:t -> exp -> cl -> cl -> unit`
5. Finalize merge
6. Start a pending merge

External scheduler:

1. Daemons (boolean propagation, congruence closure,...)
2. Decisions

- Domain change
- A class start to be used
- A semantic value is used for the first time
- A class is not anymore the representative class

- Choice: a possible decision (propositional variable in sat)
 - Booleans: propositional variable
 - Equality: equalities between uninterpreted class
 - Arithmetic: arithmetic variable
- Explanation: the constraint responsible for the modification
 - Booleans: $\neg(c_1 \vee \neg c_2 \vee c_3)$ and the kind of propagation
 - Equality: $c_1 = c_2 = c_3 = c_4$ and idem
 - Arithmetic pivot: linear combination
- Conflict:
 - Booleans: map $\{c_1 \mapsto \top; c_2 \mapsto \perp\}$
 - Equality: depend on the type
 - Arithmetic: polynomial.

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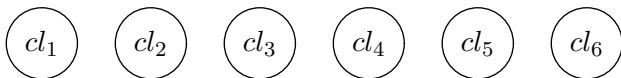
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$$x = y - 1 \wedge f(2) = t \wedge y = 3 \implies f(x) = t$$

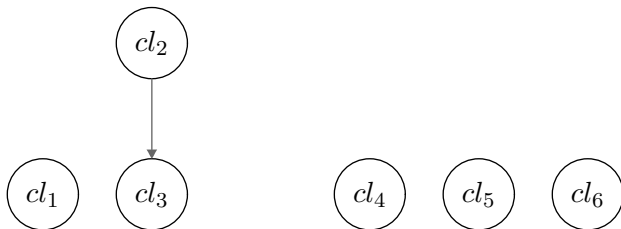
Representatives give bad explanation

$$cl_2 = cl_3, cl_5 = cl_4, cl_5 = cl_3, cl_6 = cl_5$$



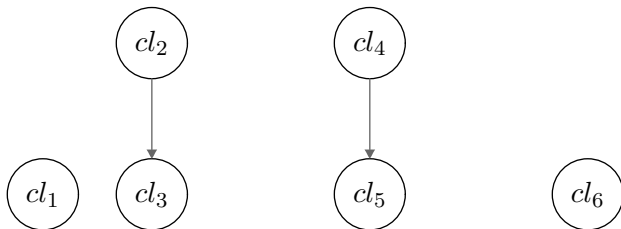
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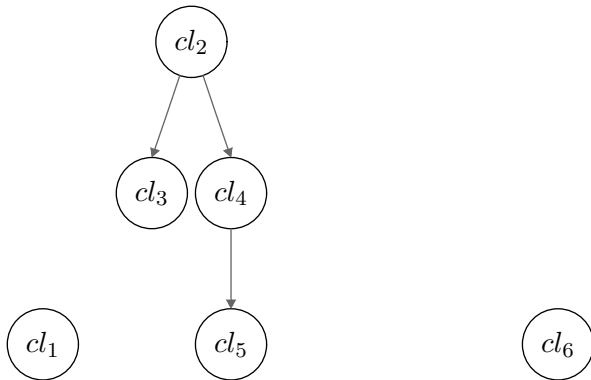
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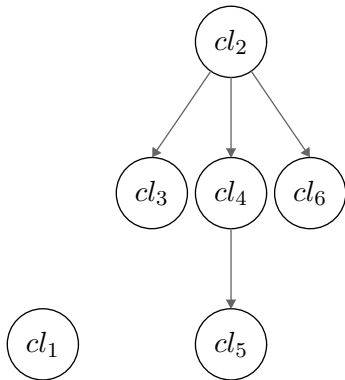
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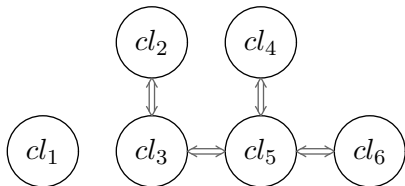
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many many many things:

1. linear arithmetic
2. quantifiers
3. bitvectors, algebraic datatypes
4. connect to Qed
5. SOPRANO
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MC-Sat implementation:

- Boolean a particular theory
- UF: generalized variable
- No sharing equivalence class
- Theoretical result

IntSat (CP 2014):

- bounded integer

